PC-255

(512) M.A./M.Sc. Mathematics (SECOND SEMESTER)

Examination- JUNE-2020

Compulsory/Optional

Group -

Paper-III

TOPOLOGY -II

Time:- Three Hours] **[Maximum Marks:80**

नोट : दोनो खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित है।

Note: Answer from Both the Section as Directed. The Figures in the right-hand margin indicated marks.

Section-A

1. Answer the following question: **1x10**

(a) Give an example of a compact space which is not Housdorff .

(b) What is Bolzano Weierstrass Property.

(c) Prove that every discrete space (X,Y) containing more than one point is disconnected.

(d) If (X,Y) is connected, then write all components of X.

(e) Define net.

(f) Define ultra-filter

(g) What is Zorn's lemma?

(h) Give an example of a directed set.

(i) Define Tychoroff cube.

(j) Define metrizable space.

2. Answer the following question: **2x5**

(a) Show that Cantor's set is compact.

(b) Let (R,U) be a topological space and EcR then prove that E is connected if E is an internal .

(c) Show that arbitrary intersection filters on a non-empty set X is a filter on X.

(d) Show that product space of two Hausdorff spaces is Hausdorff.

(e) Let (X,D) be any discrete topological space. Show that a net (f,x,A,≥) in X converges to a point $x_0 \in X$ iff f is eventually in $\{x_0\}$.

Section-B

12x5

3. (a) State and prove Heine Borel Theorem.

(b) Prove that continuous image of a compact space is compact.

OR

(a) Prove that a countably compact topological space has BWP.

(b) Show that every compact topological space is locally compact but the converse need not be true.

4. (a) Prove that a topological space X is connected iff every non-empty proper subset of X has a non-empty frontier.

(b) Let $\{C_\lambda : \lambda \in \wedge\}$ be a family of connected subsets of a space X of such that $\cap \{C_\lambda : \lambda \in \wedge\} \neq \emptyset,$ then prove that $\cup \{C_\lambda : \lambda \in \wedge\}$ is connected

OR

(a) Prove that the components B of a totally disconnected space (X,Y) are the singleton subsets of X.

(b) Prove that a topological space X is disconnected iff there exists a continuous mapping and X on to the discrete two points space $\{0,1\}$.

5. (a) Let (X,Y) be a topological space and $Y \subset X$. Then prove that Y is Y-open iff no net in X-Y can converge to a point in Y.

(b) Prove that every filter on asset X is contained in an ultra-filter on X.

OR

(a) Prove that a topological space (X,Y) is Housdorff iff every convergent filter in X has a unique limit.

(b) Late A be any non-void family of subsets of a set X. Then prove that there exists a filter an X, containing A iff A has FIP.

6. (a) Prove that a topological I space (X,Y) is Housborff iff every net in X has a unique limit.

(b) Prove that a topological space X,Y) is compact iff every net in X has a cluster point.

OR

(a) Prove that a topological space is compact iff every ultra-filter in is convergent.

(b) Let E be a connected subset of a Space X. If F is a subset of X such that $E \subset F \subset \bar{E}$ they prove that F is connected.

7. (a) Prove that the product space XxY is connected iff X and Y are connected.

(b) State and prove Alexander sub-base lemma.

OR

Stand and prove Urysohnmetrization theorem